Photon Structure Functions
beyond the SUSY Threshold

D. A. Ross and L. J. Weston

Department of Physics & Astronomy, University of Southampton,
Southampton S017 1BJ, UK

Abstract

We evolve virtual photon parton densities up to the SUSY threshold and higher using coupled inhomogeneous DGLAP differential equations. Reliable input parameterizations were available from the c-quark threshold. Limited P^2 (target photon virtuality) dependence is observed. The difference to the photon structure function is shown to be significant with the introduction of SUSY dependent splitting functions. A negligible difference is observed by letting the gluino mass enter after the squark mass. An effort is made to include the squark threshold effect in such a way that both the renormalization group equations are satisfied and the perturbative calculation is reproduced.

1 Introduction

There has recently been a great deal of interest in the structure function of the photon. This is obtained from the scattering cross-section between a highly virtual photon with large square momentum Q^2 used as a probe and a nearly on-shell target photon with square momentum P^2, $(P^2 \ll Q^2)$. If the target square momentum is also large (whilst maintaining the inequality $P^2 \ll Q^2$), the entire structure function can be calculated using renormalization group improved perturbation theory, whereas for low P^2 one is limited to a determination of the Q^2 dependence and, as in the case of deep-inelastic electron-proton scattering, one needs input information on the structure functions at some (low) value of Q^2, which cannot be determined by perturbation theory. A study of the photon structure function as a function of P^2 therefore provides information on the extrapolation between the perturbative and non-perturbative regimes of QCD.

Heuristically, one talks about separating the structure function into "direct" and "resolved" contributions. The former being exactly calculable in perturbation theory and the latter involving the uncalculable probability that the photon splits into other fundamental particles before being probed. Whereas such a picture is useful at the leading order level, higher order corrections mix these contributions. A formal and more precise analysis was first proposed by Witten [1] who pointed out that in an operator product expansion for photon-photon scattering the set of operators used in the case of photon-proton scattering must be augmented by a tower of photonic operators, whose matrix elements with the target photon are of order unity. In the (more intuitive) DGLAP approach one argues that since the probability of finding a particle other than a photon inside a photon is of order α_{em}, the probability of finding a "photon inside a photon" is unity plus corrections of order α_{em}. The DGLAP analysis must then be augmented by further off-diagonal splitting functions K_q and K_G which are interpreted as the perturbative probability for a photon to emit quark or gluon with a given fraction of its longitudinal momentum.

Interest in photon structure functions has recently been rekindled by the prospect of a future high-energy electron-positron collider with centre-of-mass energy of up to 1 TeV. Such a machine would enable an investigation of the photon structure function over a sufficiently wide range of Q^2 and P^2 to provide a stringent test of the evolution of these structure functions. Moreover, if the postulated existence of Supersymmetry (SUSY) turns out

to be vindicated, these structure functions will reflect the existence of supersymmetric partners within photons. The contribution to the structure function due to the crossing of the threshold for the production of squarks, was first calculated by Reya [6]. However, a consistent analysis of the effect of supersymmetry on the photon structure function requires a full analysis of the enlarged DGLAP formalism in which above the SUSY threshold the standard splitting functions are augmented with splitting functions involving squarks and gluinos. This paper reports on such an analysis and displays results in which it can be seen that SUSY gives rise to a measurable increase in the Q^2 evolution of the structure photon structure function above threshold. Care must be taken to ensure a consistent treatment of the threshold behaviour for squark production as one passes through the threshold and this is discussed in detail.

The outline of the paper is as follows: In Section 2 we discuss the formalism, outlining the extension of the evolution equations to the regime in which squarks and gluinos are excited. We also give a description of the threshold treatment. Section 3 displays our results obtained from numerical solution of the enlarged evolution equations. We show the dependences on the SUSY threshold and on the square momentum P^2 of the target photon as well as on the usual variables Q^2 and Bjorken-x. In Section 5 we discuss our conclusions.

2 Formalism

We follow the formalism of Glück and Reya [2]. We will initially be concerned with quark and gluon distributions up to the SUSY threshold.

The nonsinglet (T) and singlet (Σ) sectors are treated separately,

$$
\begin{aligned}
T3 &= 2(u-d) \\
T8 &= 2(u+d-2s) \\
T15 &= 2(u+d+s-3c) \\
T24 &= 2(u+d+s+c-4b) \\
T35 &= 2(u+d+s+c+b-5t) \\
\Sigma &= 2\sum_{i}^{f} q_i
\end{aligned}
$$

where u, d, s, c, b, t refer to the relevant quark distributions. The factor of 2 accounts for the anti-quark distribution since for a photon $q_i = \bar{q}_i$. f runs over all relevant quark flavours. Each quark distribution is zero at and below its threshold hence each new non-singlet (T) is equal to the singlet (Σ) at threshold.

The evolution is controlled by the following inhomogeneous DGLAP differential equations,

$$\frac{dT}{d \ln Q^2} = P_{TT} \otimes T + K_T \tag{2.1}$$

for each singlet (T) and the coupled equations,

$$\frac{d\Sigma}{d \ln Q^2} = P_{\Sigma\Sigma} \otimes \Sigma + P_{\Sigma G} \otimes G + K_\Sigma$$

$$\frac{dG}{d \ln Q^2} = P_{G\Sigma} \otimes \Sigma + P_{GG} \otimes G + K_G \tag{2.2}$$

for the singlet (Σ) and gluon (G) sector.

For each distribution $F(x, Q^2)$ above, the convolution $\otimes$ is defined as,

$$P_{ij} \otimes F_j \equiv \int_x^1 \frac{dy}{y} P_{ij}\left(\frac{x}{y}, Q^2\right) F_j(y, Q^2) \tag{2.3}$$

where $P_{ij}(x, Q^2)$ consists of the splitting functions $p_{ij}^{(0)}$ in Leading Order (LO) and $p_{ij}^{(1)}$ in next to leading order (NLO),

$$P_{ij} = \left[\frac{\alpha_s}{2\pi}\right] p_{ij}^{(0)} + \left[\frac{\alpha_s}{2\pi}\right]^2 p_{ij}^{(1)} + \cdots \tag{2.4}$$

The main difference between the evolution of the photon structure function and that of the proton structure function is the presence of the inhomogeneous K_i terms in the evolution equations. Essentially these consist of $\gamma \to$ quark and $\gamma \to$ gluon splitting functions. They appear in the evolution equations without any convolution with a parton distribution since the "photon density" inside a photon is taken to be $\delta(1-x)$ up to corrections of order α_{em}.

$$K_i = \left[\frac{\alpha_{em}}{2\pi}\right] k_i^{(0)} + \left[\frac{\alpha_{em}}{2\pi}\right]\left[\frac{\alpha_s}{2\pi}\right] k_i^{(1)} + \cdots \tag{2.5}$$

3

F_2^γ in LO is given by,

$$\frac{1}{x}F_2^\gamma = \left\{ q_{\mathrm{NS}} + \langle e^2 \rangle \Sigma \right\} \tag{2.6}$$

where

$$q_{\mathrm{NS}} = \sum_f \left(e_q^2 - \langle e^2 \rangle \right) (q_i + \bar{q}_i) \quad , \quad \langle e^k \rangle = \frac{1}{f} \sum_f e_q^k$$

$\alpha_s(Q^2)$ evolves according to

$$\frac{\alpha_s(Q^2)}{4\pi} = \frac{1}{\beta_0 \ln Q^2/\Lambda^2} - \frac{\beta_1}{\beta_0^3} \frac{\ln\left(\ln Q^2/\Lambda^2 \right)}{(\ln Q^2/\Lambda^2)^2} \tag{2.7}$$

where $\beta_0 = 11 - 2f/3$ and $\beta_1 = 102 - 38f/3$. All expressions refer to the $\overline{MS}$ renormalization scheme hence we use $\Lambda_{\overline{MS}}$ which depends f. We evolve in NLO to the t-quark threshold and then we evolve in LO thereafter. This is because we can only evolve in LO above the SUSY threshold so in order to compare F_2^γ for different values of the squark mass M_s we must evolve in the same way from the t-quark threshold. Quark masses are taken as $M(c) = 1.5\,GeV$, $M(b) = 4.5\,GeV$, $M(t) = 174\,GeV$.

Our input data were parameterizations [3] of virtual photon parton densities taken at the c-quark threshold. A c-quark mass of $1.5\,GeV$ limits P^2 to less than $1.8\,GeV$ which gives a small ratio $r = P^2/Q^2 \simeq 10^{-6}$ at high Q^2. We could not find reliable parameterizations at higher Q^2 that were dependent on P^2.

In order to evolve to the SUSY threshold we use LO and NLO splitting functions [4] in (Eq. 2.4) and inhomogeneous terms [2] in (Eq. 2.5).

Above the SUSY threshold M_s we are also concerned with squark and gluino distributions. As before we have a nonsinglet (S) and singlet (Γ) sector,

$$
\begin{aligned}
S3 &= 4(Su - Sd) \\
S8 &= 4(Su + Sd - 2Ss) \\
S15 &= 4(Su + Sd + Ss - 3Sc) \\
S24 &= 4(Su + Sd + Ss + Sc - 4Sb)
\end{aligned}
$$

4

$$S35 = 4(Su + Sd + Ss + Sc + Sb - 5St)$$

$$\Gamma = 4 \sum_{i}^{f} S\bar{q}_i$$

where Su, Su, Ss, Sc, Sb, St refer to the squark distributions. The factor of 4 arises because $Sq_i^R = Sq_i^L = S\bar{q}_i^R = S\bar{q}_i^L$. For simplicity all squark distributions start at zero at the SUSY threshold M_s, although we could introduce them in steps as before with the quarks. The gluino distribution starts at zero at the gluino threshold M_g.

The evolution is controlled by the following inhomogeneous DGLAP differential equations. Each set of nonsinglets are coupled ie. T3 with S3, T8 with S8, etc...

$$\frac{dT}{d\ln Q^2} = P_{TT} \otimes T + P_{TS} \otimes S + K_T$$

$$\frac{dS}{d\ln Q^2} = P_{ST} \otimes T + P_{SS} \otimes S + K_S \tag{2.8}$$

Given that in general the gluino mass M_g is greater than the squark mass M_s the nonsinglet sector evolution is given in the region $M_g^2 \geq Q^2 \geq M_s^2$ by,

$$\frac{d\Sigma}{d\ln Q^2} = P_{\Sigma\Sigma} \otimes \Sigma + P_{\Sigma G} \otimes G + P_{\Sigma\Gamma} \otimes \Gamma + K_\Sigma$$

$$\frac{dG}{d\ln Q^2} = P_{G\Sigma} \otimes \Sigma + P_{GG} \otimes G + P_{G\Gamma} \otimes \Gamma + K_G$$

$$\frac{d\Gamma}{d\ln Q^2} = P_{\Gamma\Sigma} \otimes \Sigma + P_{\Gamma G} \otimes G + P_{\Gamma\Gamma} \otimes \Gamma + K_\Gamma \tag{2.9}$$

and in the region $Q^2 \geq M_g^2$ by,

$$\frac{d\Sigma}{d\ln Q^2} = P_{\Sigma\Sigma} \otimes \Sigma + P_{\Sigma G} \otimes G + P_{\Sigma\Gamma} \otimes \Gamma + P_{\Sigma L} \otimes L + K_\Sigma$$

$$\frac{dG}{d\ln Q^2} = P_{G\Sigma} \otimes \Sigma + P_{GG} \otimes G + P_{G\Gamma} \otimes \Gamma + P_{GL} \otimes L + K_G$$

5

$$\frac{d\Gamma}{d\ln Q^2} \;=\; P_{\Gamma\Sigma}\otimes\Sigma \;+\; P_{\Gamma G}\otimes G \;+\; P_{\Gamma\Gamma}\otimes\Gamma \;+\; P_{\Gamma L}\otimes L \;+\; K_\Gamma$$

$$\frac{dL}{d\ln Q^2} \;=\; P_{L\Sigma}\otimes\Sigma \;+\; P_{LG}\otimes G \;+\; P_{L\Gamma}\otimes\Gamma \;+\; P_{LL}\otimes L \;+\; K_L \qquad (2.10)$$

where L is the gluino distribution. In analogy with the K_i used below the SUSY threshold, K_Γ and K_L are the splitting functions of a photon to a squark and gluino respectively. In the limit where the gluino mass M_g is taken to be equal to the squark mass M_s we do not need (Eqs. 2.9).

The $P_{ij}(x, Q^2)$ are now a different SUSY set of LO splitting functions [5]. However in order to reproduce the squark threshold condition,

$$Q^2\frac{(1 - x - r\,x)}{x} \;\geq\; 4M_s^2 \qquad (2.11)$$

we use the SUSY set if this applies or the standard set if it does not. Importantly this now means that the threshold depends both on x and Q^2. This is possible because a convolution (Eq. 2.3) only feeds each distribution in the region $y \geq x$. Hence for a particular Q^2, SUSY splitting functions are used below a certain value of x and standard splitting functions are used above this x during each convolution.

Our choice of LO inhomogeneous terms (Eq. 2.5) is made in order to treat the squark threshold in a meaningful way.

The tree level squark contribution to F_2^γ (Appendix A) is important in determining the $\gamma \to$ squark splitting function i.e., the squark inhomogeneous term in (Eqs. 2.8, 2.9, 2.10). The choice is dependent on the squark threshold condition (Eq. 2.11), which is function of both x and Q^2. At a particular Q^2 there will be a region $x > x_s$ where squarks cannot be produced.

The contribution to F_2^γ from squark production (Eq. A.1) is obtained from ordinary perturbation theory and includes a term proportional to $\ln(Q^2/4M_s^2)$, which is already accounted for as K_Γ, by the use of the full SUSY set of splitting functions discussed above. In order to reproduce the correct renormalization group improved evolution far above the threshold we subtract this term (apart from terms proportional to M_s^2/Q^2 which are negligible far above the threshold) from (Eq. A.1) and introduce the contribution as SB_γ in (Eq. 2.12).

Above the SUSY threshold F_2^γ is then obtained from

$$\frac{1}{x}F_2^\gamma = \left\{ q_{\text{NS}} + \langle e^2 \rangle \Sigma \right\} + \left\{ Sq_{\text{NS}} + \langle e^2 \rangle S\Sigma \right\} + 2 \times 3f \langle e^4 \rangle \left[\frac{\alpha_{em}}{4\pi} \right] SB_\gamma \quad (2.12)$$

(the factor of 2 accounts for right- and left-handed squarks).

The SB_γ term is (up to an overall constant) the contribution to F_2^γ from squark production, with the removal of the above-mentioned term proportional to $\ln(Q^2/M_s^2)$. i.e.

$$SB_\gamma \quad \propto \quad F_{2,squark}^\gamma - 2x(1-x) \ln(Q^2/4M_s^2), \qquad (2.13)$$

We note that the difference between using (Eq. A.1) and (Eq. A.2) is negligible in our case because we are limited to $P^2 < 1.8\,GeV^2$ at the c-quark threshold, giving an $r \simeq 10^{-6}$ above the SUSY threshold.

However this is a different way of treating the threshold behaviour from that in [2]. At $Q^2 \gg 4M_s^2$ it satisfies the Renormalization Group equations since the dominant part is in the inhomogeneous term. In the region $Q^2 \simeq 4M_s^2$ this approach will reproduce the perturbative calculation with the correct threshold behaviour up to $(\ln(Q^2/4M_s^2))^2$. There should of course be a small mismatch at large Q^2 and large x. However we have eradicated this by coding the threshold behaviour into each convolution as a choice of SUSY or non-SUSY splitting functions. The problem left is that of the discontinuity at the threshold x boundary. An equivalent problem is discussed in [7] involving the DIS_γ factorization scheme. Perturbative instabilities mean we would have to go to NLO and eliminate the SB_γ term by altering what would be our NLO inhomogeneous terms (Eq. 2.5). However NLO SUSY splitting functions are beyond the scope of the present work.

Finally it should be noted how quickly F_2^γ changes away from the threshold with decreasing x. In (Eq. A.1), the term

$$v = [1 - 4M_s^2 x/Q^2(1-x)]^{1/2}$$

moves rapidly away from zero in decreasing x meaning that the coefficients of

$$\ln\left(\frac{1+v}{1-v}\right) \quad \text{and} \quad v$$

in (Eq. A.1) give rise to a real threshold contained in the term SB_γ in the perturbatively stable x region.

7

To summarize, we take parameterizations of quark and gluon distributions inside a virtual photon at the c-threshold. Using DGLAP inhomogeneous differential equations we evolve the relevant non-singlet, singlet and gluon distributions up to the SUSY threshold. From here we run the distributions separately, including or not, the effects of squarks and gluinos. At some $\sqrt{Q^2}$ we form F_2^γ for the virtual photon in such a way as to take account of the SUSY threshold condition.

3   Results

The variable parameters of the evolution are the P^2 (target virtuality), M_s (squark mass), M_g (gluino mass), Q^2 (incident virtuality) and Bjorken x. We took these in the ranges,

$$0 \leq \sqrt{P^2} \leq 1300 \, MeV$$

$$175 \, GeV \leq M_s \leq 300 \, GeV$$

$$200 \, GeV \leq M_g \leq 300 \, GeV$$

$$700 \, GeV \leq \sqrt{Q^2} \leq 1500 \, GeV$$

and in all cases F_2^γ/α_{em} is actually plotted.

Figure 1 shows a generalised evolution to $1000 \, GeV$.

The base reference is at $175 \, GeV$. There is a considerable difference with the SUSY mixing. Note that allowing the gluino mass to be greater than the squark mass produces a negligible effect. Note also that the evolution graphs coincide above the squark threshold condition (Eq. 2.11), this being due to it being encoded into each convolution.

The discontinuity at the threshold can be seen. This results in a sudden drop in F_2^γ below the control non-SUSY graph, this being due to the perturbative instabilities already discussed.

It is important to note the rapidity with which F_2^γ increases as x decreases away from the threshold. This effect due to the SB_γ term in the perturbatively stable region is discussed in section 2.

Figure 2 shows P^2 dependence up to $1300 \, MeV$. From here on we plot $M_g = M_s$ since we have shown the $M_g > M_s$ difference to be negligible.

8

We are limited by our parameterizations in that they are restricted in P^2 at the c-quark threshold. However non-negligible differences can be noted in low x even at $\sqrt{P^2} = 1300\,MeV$.

Figure 3 shows M_s dependence between $175\,GeV$ and $300\,GeV$.

The base graph is without the SUSY contributions. As expected, as the squark mass M_s increases the distribution's approach to the non-SUSY limit. Also the thresholds move to lower x as the threshold condition (Eq. 2.11) requires higher Q^2 to produce squarks of higher mass.

Figures 4 and 5 show how F_2^γ varies with $\sqrt{Q^2}$ at two fixed values of x. All graphs show how the distributions must approach the non-SUSY distribution for high M_s. However for $x = 0.66$ we can see the gradual approach to a threshold in increasing M_s. For $M_s = 275\,GeV$ it is evident that for low Q^2 squarks cannot be produced and the distribution coincides with the non-SUSY distribution. Then apart from the perturbative instability the distribution rises in higher Q^2.

Comparative Evolution for $\sqrt{P^2} = 1.3$ GeV, $M_s = 175$ GeV, $M_g = 175, 300$ GeV, $\sqrt{Q^2} = 1000$ GeV
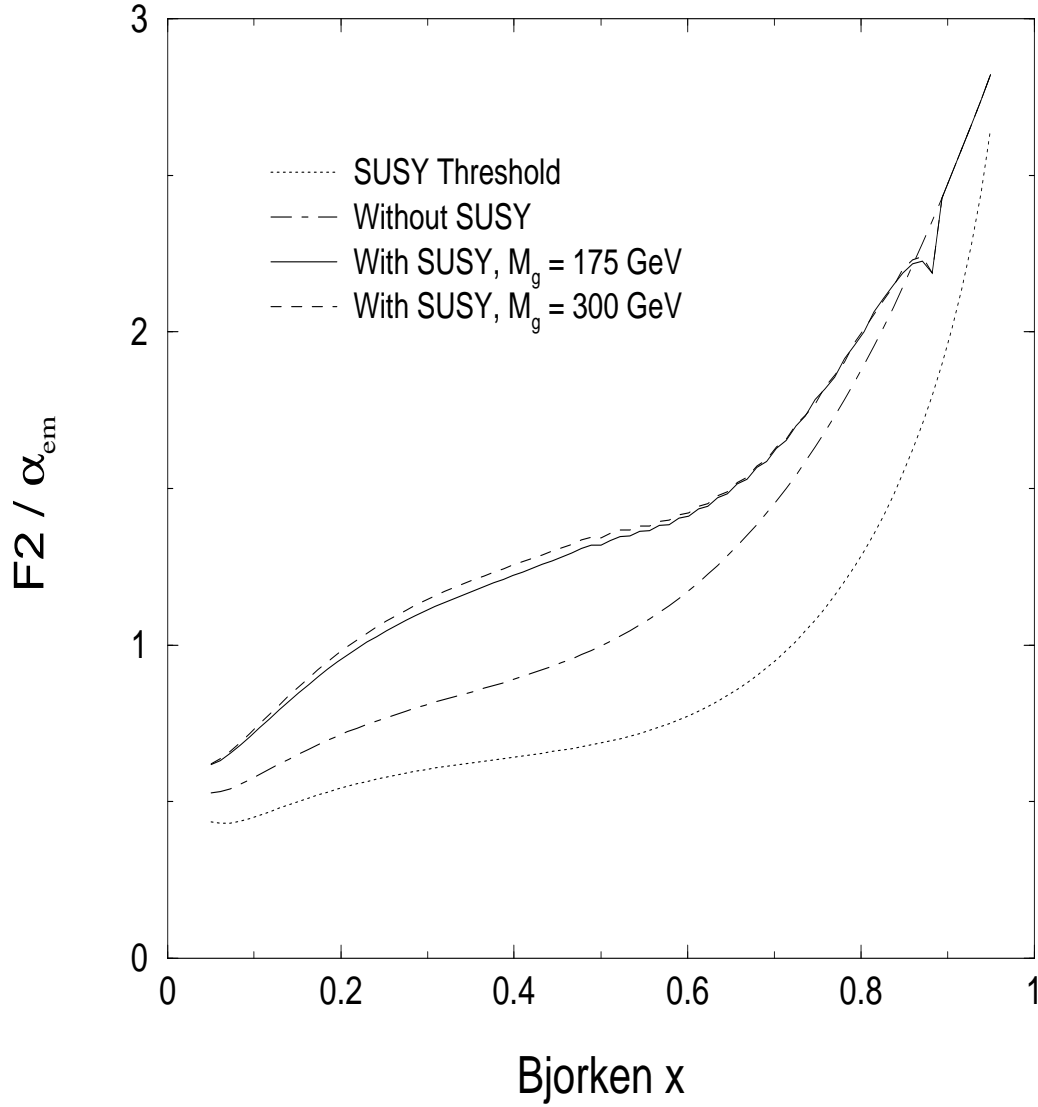

Figure 1: Comparative Evolution of Structure Function with and without SUSY splitting functions. Difference due to a higher gluino mass M_g is negligible.
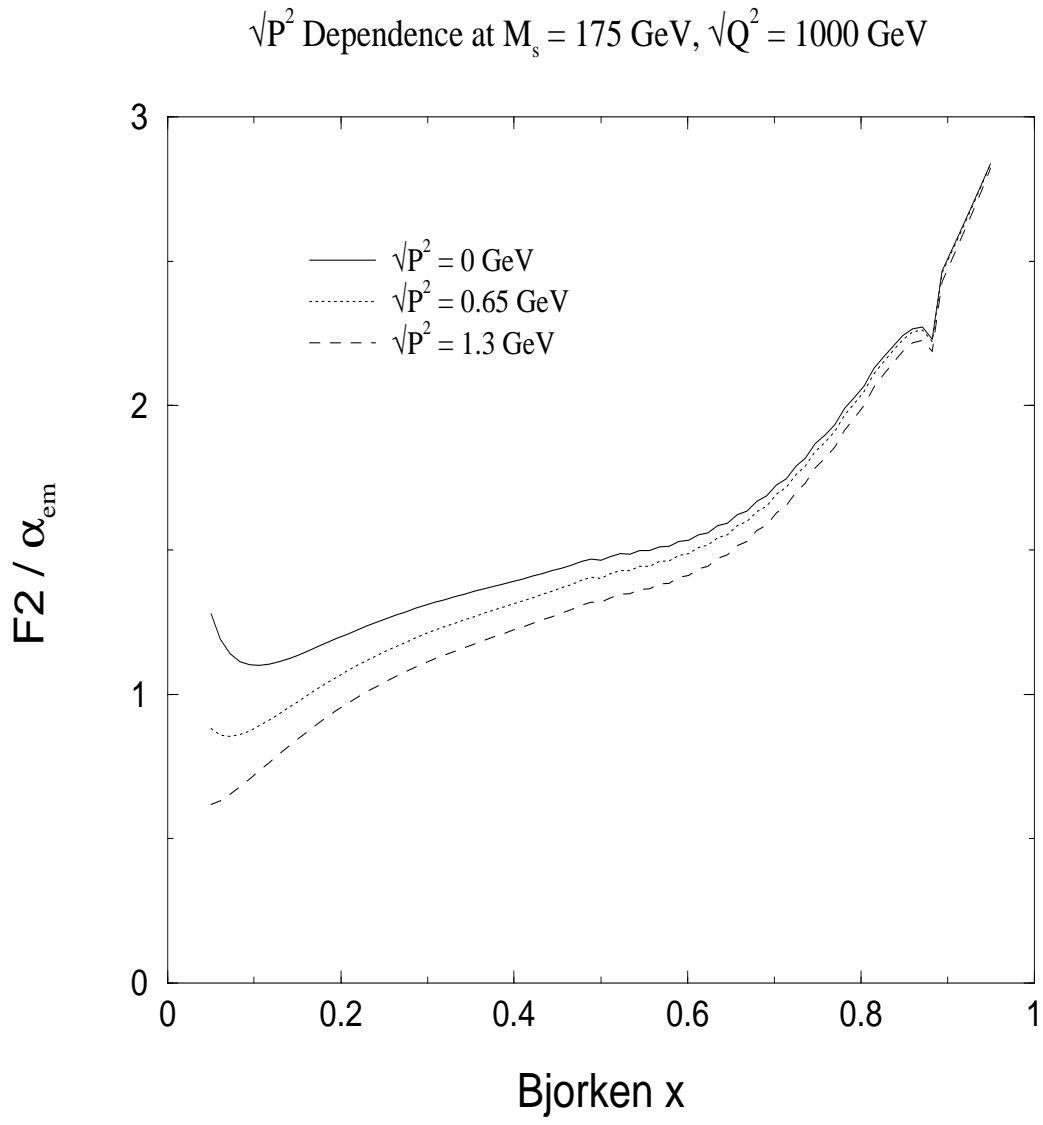
$\sqrt{P^2}$ Dependence at M_s = 175 GeV, $\sqrt{Q^2}$ = 1000 GeV

Legend:
- $\sqrt{P^2}$ = 0 GeV
- $\sqrt{P^2}$ = 0.65 GeV
- $\sqrt{P^2}$ = 1.3 GeV

Y-axis: F2 / α_{em}

X-axis: Bjorken x

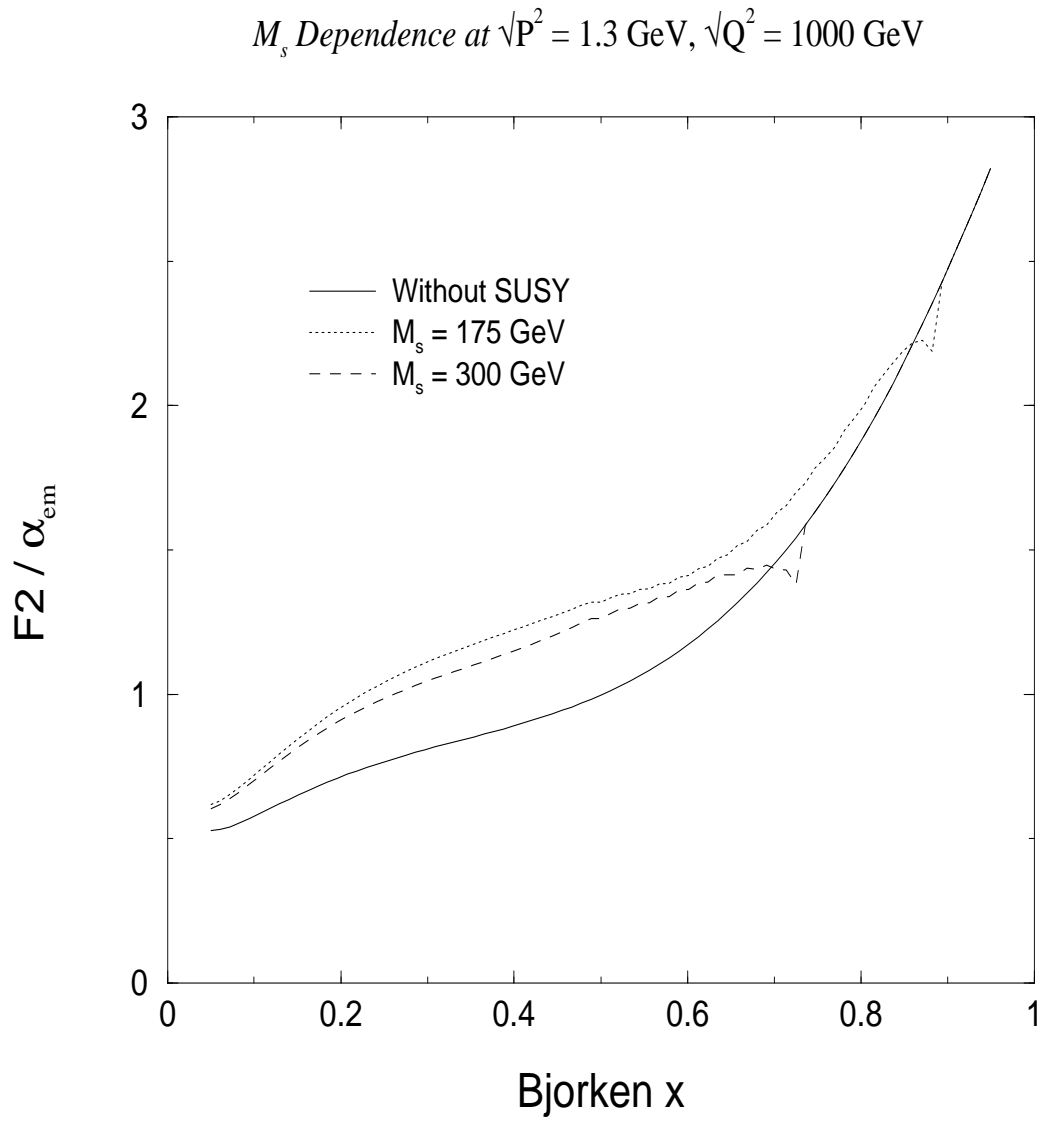Figure 2: $\sqrt{P^2}$ dependence of structure function for fixed squark mass M_s at a fixed probe virtuality $\sqrt{Q^2}$

Figure 3: Dependence of structure function on squark mass M_s at a fixed target virtuality $\sqrt{P^2}$ and probe virtuality $\sqrt{Q^2}$
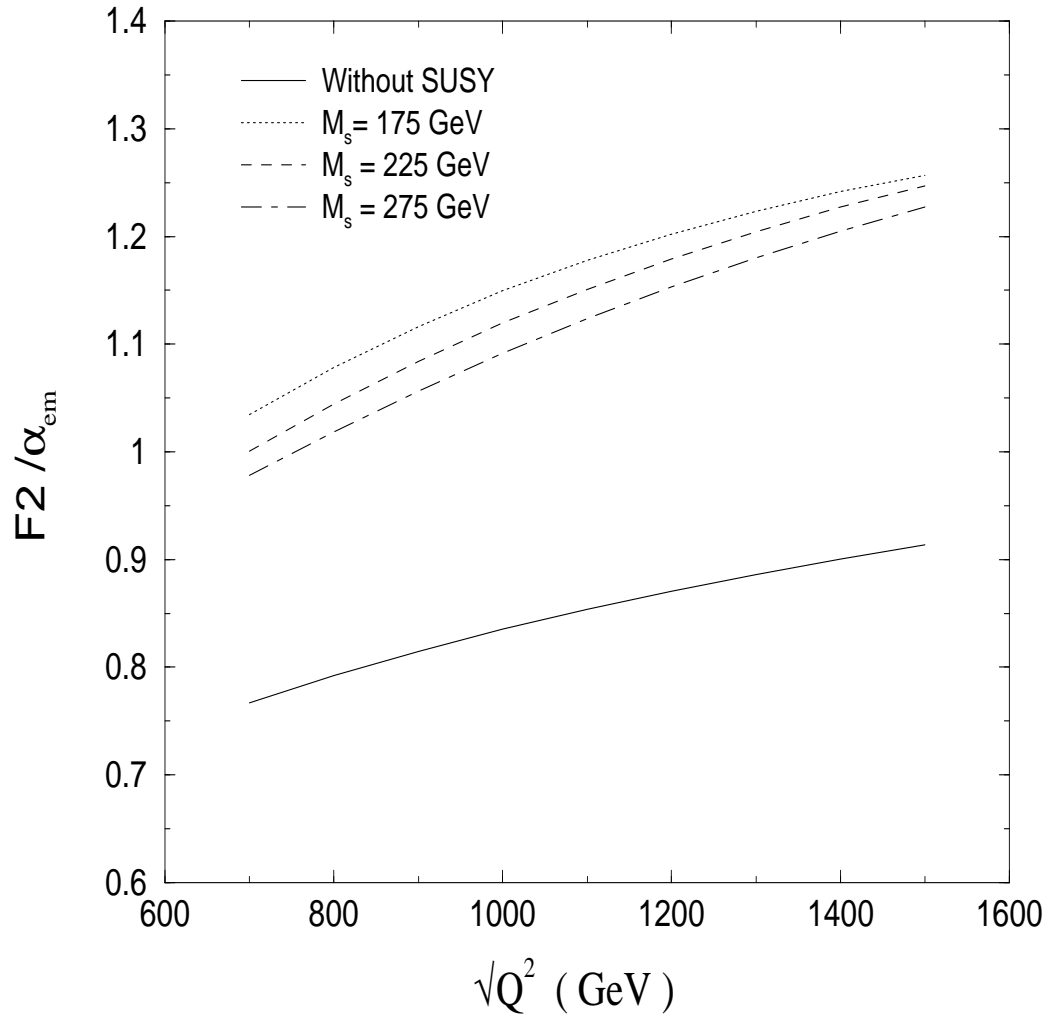
Q^2 *Dependence for fixed x = 0.33*

Figure 4: $x = 0.33$
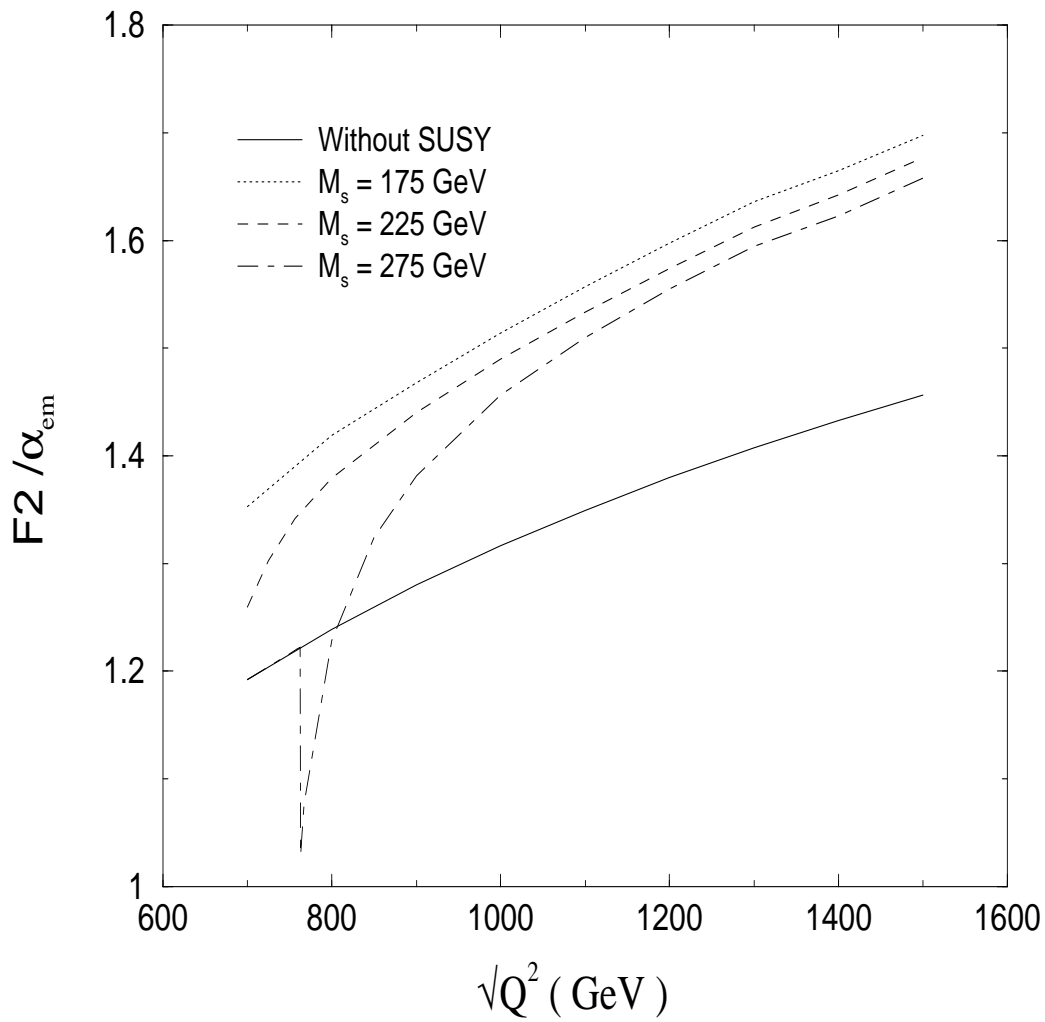
Q^2 *Dependence for fixed x = 0.66*

Figure 5: $x = 0.66$

14

4 Conclusions

We see from Figs. 1-5 that if one can build a machine for which values of Q^2 approach 1 TeV^2 (about twice the squark production threshold) there is a substantial increase in the value of F_2^γ for the photon. Indeed, the evolution between the SUSY threshold and 1 TeV is more than doubled if SUSY particles, taken to have a mass of 175 GeV, are present. The difference between the structure functions with and without SUSY in the middle range of Bjorken-x is over 30%, which should be easily detectable.

The effect at $Q^2 = 1$ TeV^2 is, of course, diminished if the SUSY threshold is increased. However, we note that taking the squark masses to 300 GeV only has a small effect on F_2^γ. Conversely, if the squark masses turn out to be substantially lighter than 175 GeV [1] , (which is not currently ruled out), there would be a significant effect on the structure functions at values of Q^2 significantly below 1 TeV^2.

The effect also diminishes if the target photon is off-shell, as will usually be the case. However, we see from Figure 2 that this effect is modest.

The results are fairly insensitive to the mass of the gluino. This is not surprising as the gluino contributes very indirectly - it can only be produced by a secondary emission from the target photon and then only probed through a further interaction with squarks. Taking the mass of the gluino below that of the squark, would have had a negligible effect as it is clear that it is the squark threshold and not the gluino threshold that must be crossed before there is any effect on the photon structure function.

In summary, we see that the effect of SUSY on the photon structure function provides a further good reason for designing a large electron-positron collider that would be capable of reaching values of Q^2 above the SUSY threshold for the middle range of Bjorken-x.

Appendix A

The contribution to F_2^γ of a left or right handed squark in deep inelastic scattering on a photon has been calculated [6],

---

[1] The lowest value we take for the squark mass is 175 GeV since this is above the threshold for t-quark and we find it useful to make comparisons of the evolution of the structure function in the presence of SUSY with that without SUSY but with six active flavours.

$$F_{2,q}^\gamma \ = \ 3e_q^4 \frac{\alpha}{\pi} x \left\{ \left[2x(1-x) + \tau x(3x-1) + \frac{1}{2}\tau^2 x^2 \right] \ln\left(\frac{1+v}{1-v}\right) \right.$$

$$\left. + \ [1 - 8x(1-x) + \tau x(1-x)]\, v \ \right\} \tag{A.1}$$

where,

$$\tau \ = \ 4M_s^2/Q^2$$

$$v \ = \ [1 - \tau x/(1-x)]^{1/2}$$

We calculated this expression for the case $P^2 \neq 0$, where $r = P^2/Q^2$. The above relation is recovered on $r \to 0$ with,

$$F_{2,q}^\gamma \ = \ 3e_q^4 \frac{\alpha}{\pi} x \left\{ B(M_s^2/Q^2)^2(1/FG)(16x^2) \right.$$

$$+ \ B(M_s^2/Q^2)(1/FG)(-48x^4 r^2 + 48x^3 r + 4x^2 r - 8x^2)$$
$$+ \ B(1/FG)(-12x^4 r^3 + 12x^3 r^2 - 2x^2 r)$$
$$+ \ B(1 - 6x^2 r + 6x^2 - 6x)$$
$$+ \ \ln(F/G)(M_s^2/Q^2)^2(B/\eta)(8x^2/b)$$
$$+ \ \ln(F/G)(M_s^2/Q^2)(B/\eta)(24x^4 r^2/b + 2x^2 r/b + 12x^2 - 2x/b - 2x)$$
$$+ \ \ln(F/G)(B/\eta)\Big(\frac{1}{2} + 6x^4 r^3/b + 12x^4 r^2/b - 12x^3 r^2/b$$
$$-12x^3 r/b + 11x^2 r/b - 3x^2 r + 4x^2/b - 6x^2 + \frac{1}{2}xr/b$$
$$\left. -\frac{1}{2}xr - 3x/b + 5x - \frac{1}{2b}\Big) \ \right\} \tag{A.2}$$

where,

$$b \ = \ 1 - 2xr$$

$$F \ = \ 1 + \eta(1 - 2xr)$$

$$G = 1 - \eta(1 - 2xr)$$

$$B = \sqrt{1 - \frac{4M_s^2 x}{Q^2(1 - x - xr)}}$$

$$\eta = \frac{B}{b}\sqrt{1 - 4x^2 r}$$

These equations are important in determining the $\gamma \rightarrow$ squark splitting function i.e., the squark inhomogeneous K_S and K_Γ . Also in determining the extra SB_γ term in (Eq. 2.12). The choice is dependent on the squark threshold condition (Eq. 2.11), which is function of both x and Q^2. At a particular Q^2 there will be a region $x > x_t$ where squarks cannot be produced.

References

[1] E.Witten, *Nucl. Phys.* **B120**, 189 (1977)

[2] M. Glück and E. Reya, *Phys. Rev.* **D28**, 2749 (1983)

[3] M. Glück, E. Reya and M. Stratmann, *Phys. Rev.* **D54**, 5515 (1996)

[4] R. Ellis, W. Stirling and B. Webber,
QCD and Collider Physics. pg 109-113
Cambridge University Press, 1996

[5] C. Kounnas and D. Ross, *Nucl. Phys.* **B214**, 317 (1983)
S.K. Jones and C.H. LLewellyn-Smith, *Nucl. Phys.* **B217**, 145 (1983

[6] E. Reya, *CERN preprint* **TH.3504**, (1983)

[7] M. Glück, E. Reya and A. Vogt, *Phys. Rev.* **D45**, 3986 (1991)